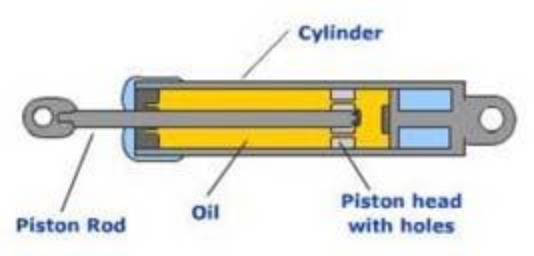
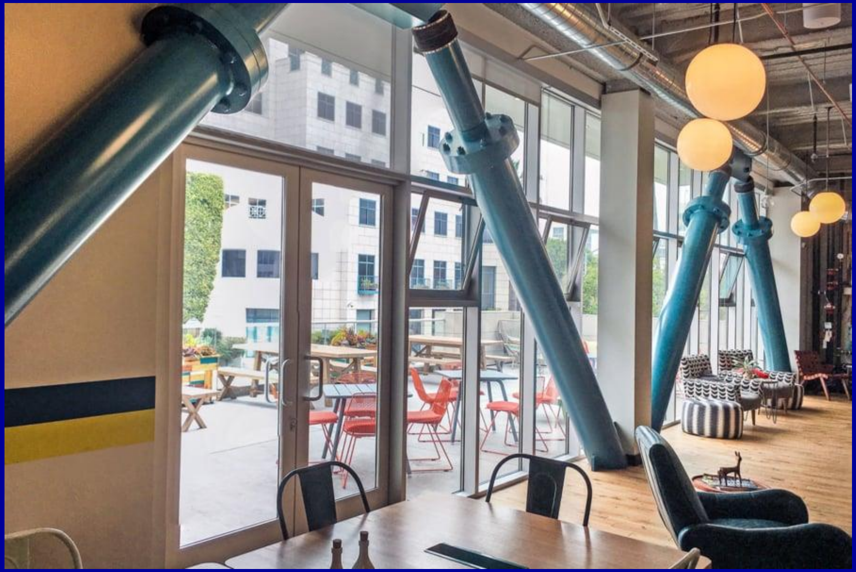
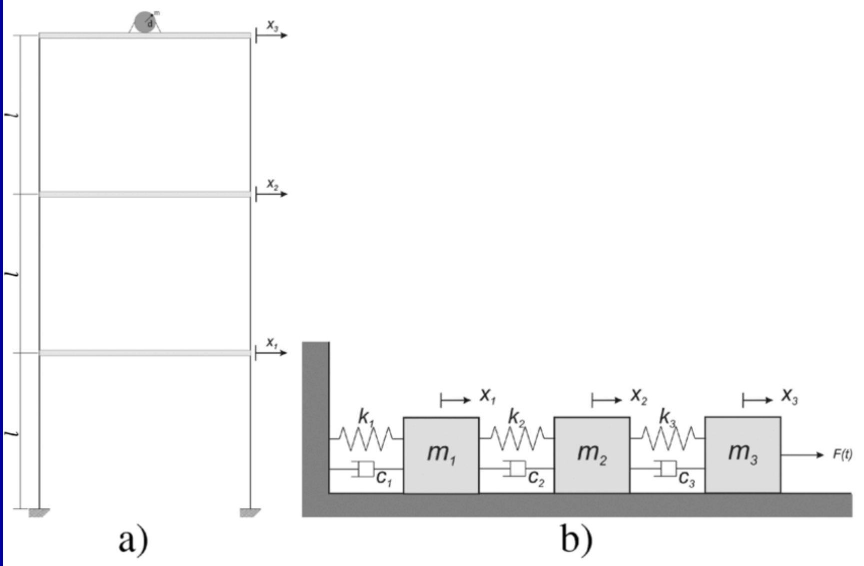
Prigušeno harmoničko titranje: drukčija točka gledišta







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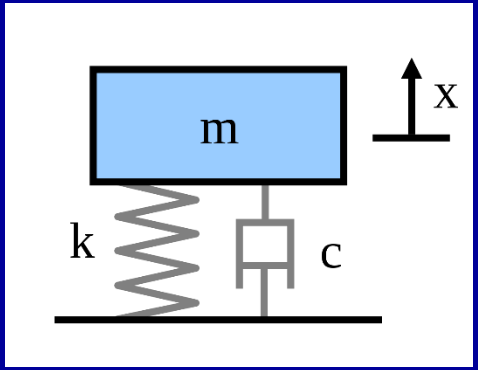
Slobodne oscilacije prigušenog sistema s jednim stupnjem slobode

$$m\ddot{x}(t) = -c\dot{x}(t) - kx(t)$$

$$\ddot{x}(t) + 2\gamma\dot{x}(t) + \omega_0^2 x(t) = 0$$

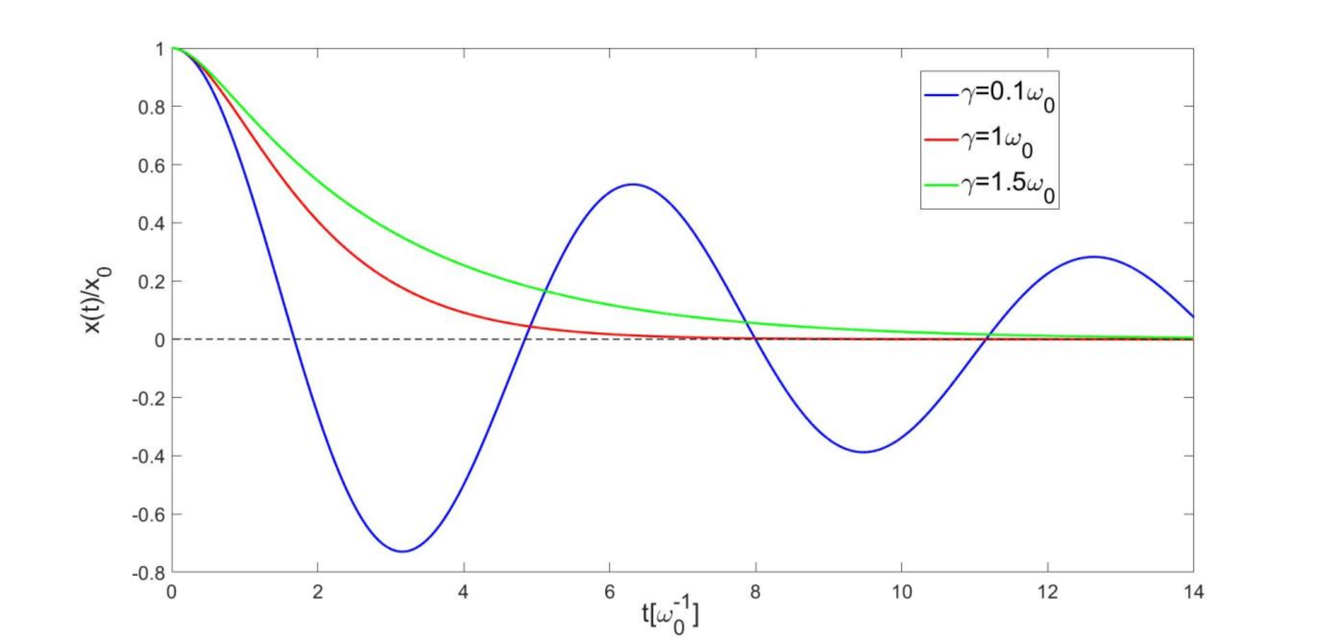


BEZ PRIGUŠENJA $\gamma = 0\ ,\ x(t) = A\cos(\omega_0 \cdot t + \phi)$

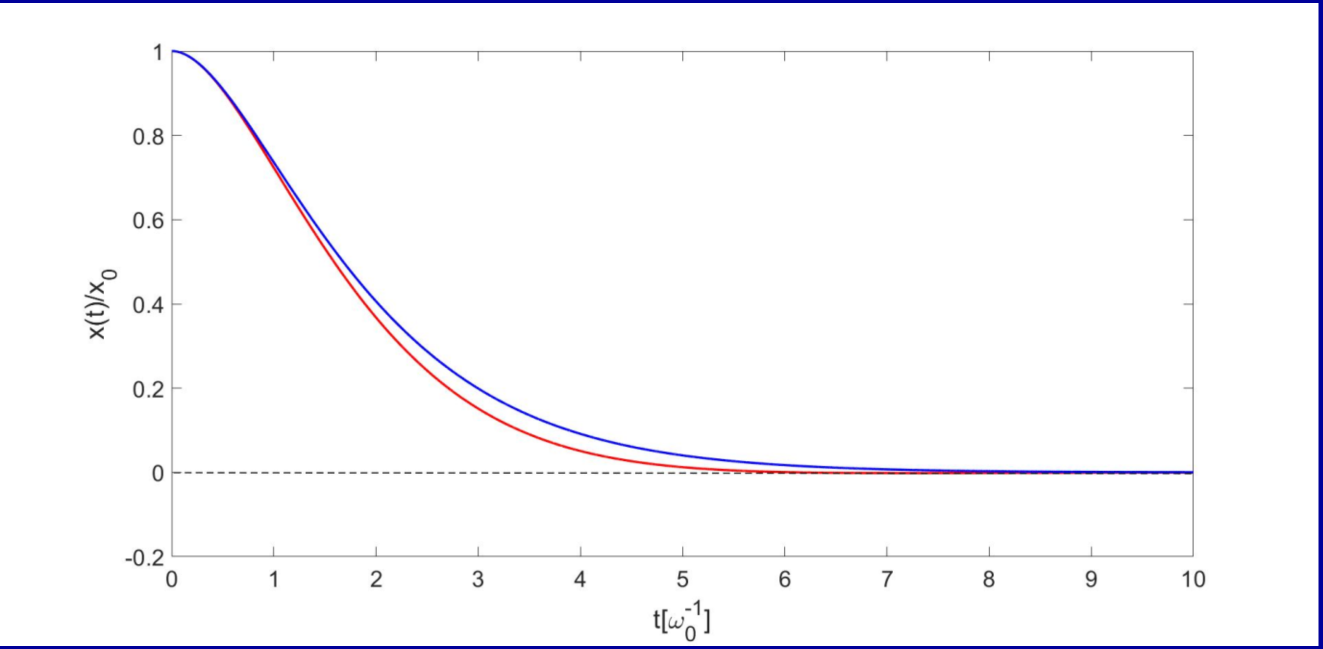
PODKRITIČNO GUŠENJE $\gamma < \omega_0\ ,\ x_{pk}(t) = A_{pk}e^{-\gamma \cdot t}\cos(\omega \cdot t + \phi_{pk})\ ,\ \omega = \sqrt{\omega_0^2 - \gamma^2}$

KRITIČNO GUŠENJE $\gamma = \omega_0\ ,\ x_k(t) = (A_k + B_k \cdot t)e^{-\omega_0 \cdot t}$

NADKRITIČNO GUŠENJE $\gamma > \omega_0\ ,\ x_{nk}(t) = A_{nk}e^{(\Omega-\gamma)\cdot t} + B_{nk}e^{(-\Omega-\gamma)\cdot t}\ ,\ \Omega = \sqrt{\gamma^2 - \omega_0^2}$



γ=0.1ω0
γ=1ω0
γ=1.5ω0
x(t)/x0
t[ω0-1]



x(t)/x0
t[ω0-1]

Kritično gušenje u literaturi

"The return to zero in a critically damped system is reached in minimum time.“

H. J. Pain, *The Physics of Vibrations and Waves* (John Wiley & Sons, Chichester, UK, 2005)

“If we look at the system for different values of damping coefficient, then critical damping is the case where the motion converges to zero in the quickest way."

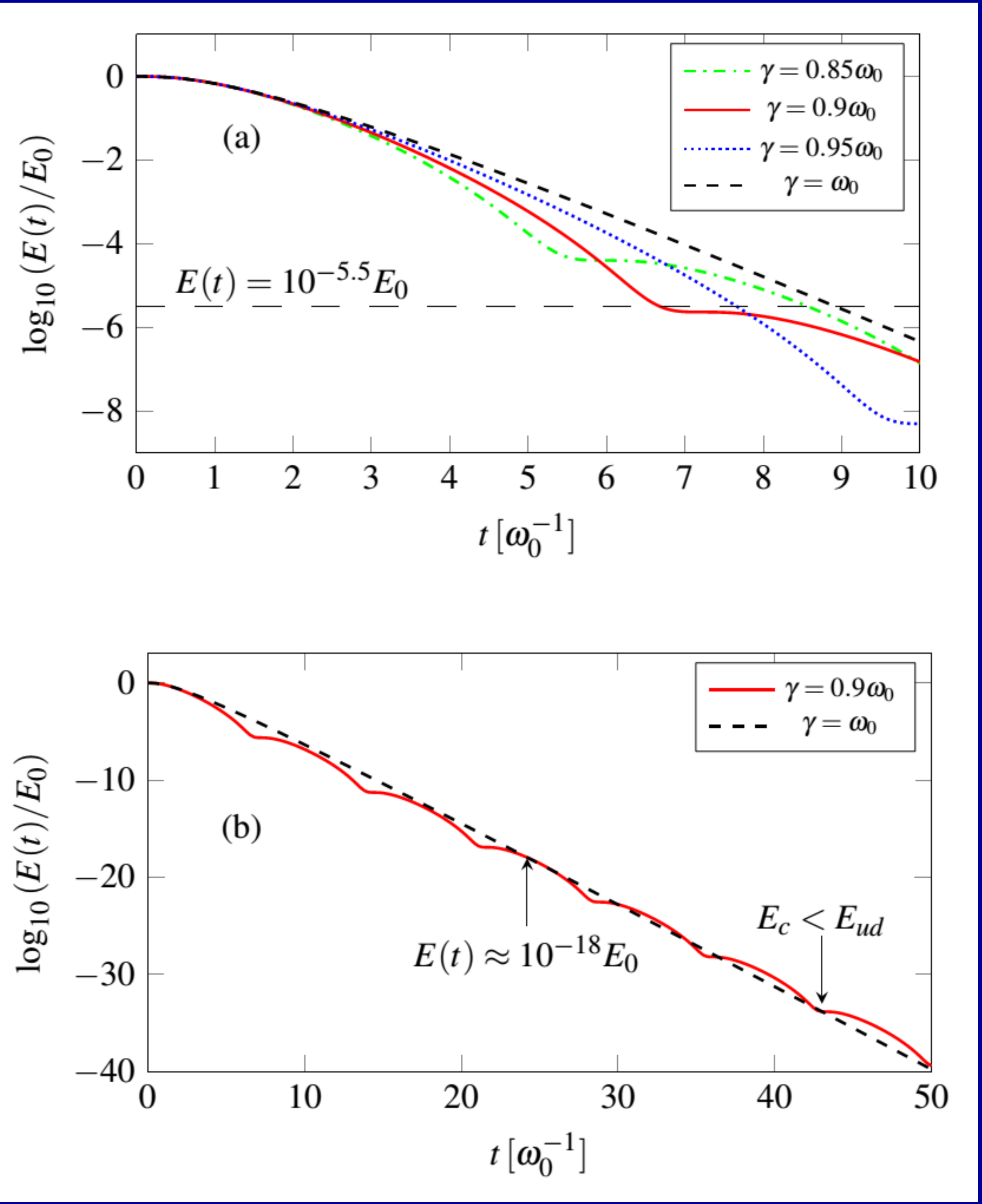
D. J. Morin, *Introduction to classical mechanics: with problems and solutions* (Cambridge University Press, Cambridge, UK, 2008)

"Critically-damped: the pendulum returns to equilibrium as quickly as it can. If the damping parameter were made slightly more or slightly less, it would result in the pendulum returning slower to its equilibrium position.“

M. Mongelli and N. A. Batista, “A swing of beauty: Pendulums, fluids, forces, and computers,” Fluids 5, 1–35 (2020).

"The critically damped system undergoes the fastest energy dissipation.“

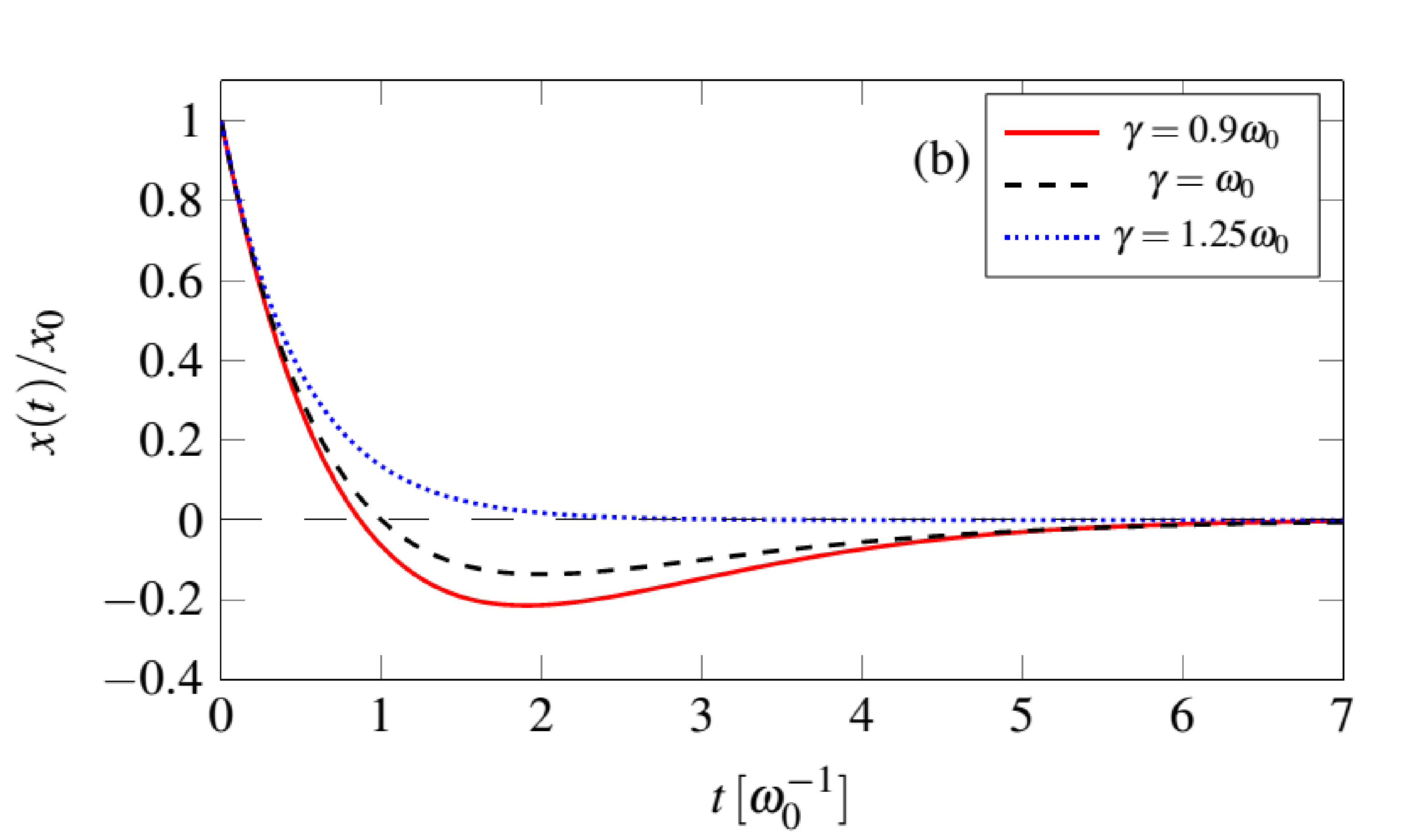
L. F. Yang, R. Arora, V. Braverman, and T. Zhao, “The Physical Systems Behind Optimization Algorithms”, 2018.



(a)
γ = 0.85ω0
γ = 0.9ω0
γ = 0.95ω0
γ = ω0
E(t) = 10^−5.5 E0
log10(E(t)/E0)
t [ω0^−1]
(b)
γ = 0.9ω0
γ = ω0
E(t) ≈ 10^−18 E0
Ec < Eud
log10(E(t)/E0)
t [ω0^−1]

$$\exp\left[-2\frac{\gamma}{\sqrt{\omega_0^2-\gamma^2}}\left(\frac{\pi}{2}+\arctan\frac{\gamma}{\sqrt{\omega_0^2-\gamma^2}}\right)\right]=10^{-\delta}$$

$E(t)/E_0$	$\gamma\,[\omega_0]$	$\tau_{A_1}\,[\omega_0^{-1}]$	$\tau_c\,[\omega_0^{-1}]$	$(\tau_c-\tau_{A_1})/\tau_c\,[\%]$
10^{-4}	0.8688	5.30	6.96	23.85
10^{-6}	0.9286	7.44	9.56	22.17
10^{-8}	0.9555	9.63	12.10	20.35
10^{-10}	0.9698	11.87	14.57	18.53
10^{-12}	0.9782	14.12	17.03	17.09
10^{-14}	0.9835	16.36	19.46	15.98
10^{-16}	0.9872	18.69	21.88	14.54
10^{-18}	0.9897	20.94	24.28	13.76



$$\gamma > \omega_0 \ , \ x_{nk}(t) = A_{nk}e^{(\Omega-\gamma)\cdot t} + B_{nk}e^{(-\Omega-\gamma)\cdot t} \ , \ \Omega = \sqrt{\gamma^2 - \omega_0^2}$$

Zaključak

1. Protivno široko zastupljenom mišljenju u literaturi, kritično gušenje nikad nije optimalan izbor ukoliko želimo da se sistem što brže vrati u ravnotežno stanje, odnosno da sistem u što kraće vrijeme izgubi početnu energiju.

2. Za većinu početnih uvjeta može se odrediti podkritični koeficijent gušenja za koji sistem najbrže dolazi u ravnotežno stanje. Izuzetak su početni uvjeti za koje je početna kinetička energija veća od početne potencijalne energije i za koje su predznaci početnog pomaka i početne brzine različiti. Za te početne uvjete može se odrediti optimalni koeficijent gušenja u nadkritičnom području.

Puno više detalja se može naći u nedavno objavljenom radu:

Damped harmonic oscillator revisited: The fastest route to equilibrium

K. Lelas, N. Poljak, D. Jukić

American Journal of Physics, 2023.